

UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 2ND FEBRUARY 2006

Organised by the **United Kingdom Mathematics Trust**
from the **School of Mathematics, University of Leeds**



RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 11 or below.
Candidates in Scotland must be in S4 or below.
Candidates in Northern Ireland must be in School Year 12 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
Six marks are awarded for each correct answer to Questions 16-25.
Each incorrect answer to Questions 16-20 loses 1 mark.
Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. The UK IMC is about solving interesting problems, not about lucky guessing.

The UKMT is a registered charity

<http://www.ukmt.org.uk>

1. What fraction is half-way between $\frac{1}{4}$ and $\frac{1}{6}$?

A $\frac{1}{10}$

B $\frac{2}{9}$

C $\frac{5}{24}$

D $\frac{3}{14}$

E $\frac{7}{12}$

2. The diagram shows seven metal rings linked together. What is the smallest number of rings that need to be cut in order to separate all the rings?

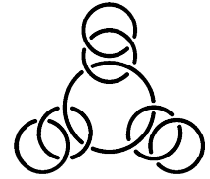
A 2

B 3

C 4

D 5

E 6



3. Which of the following is not prime?

A $2^2 - 1$

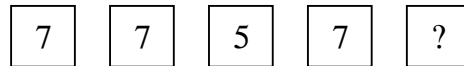
B $2^3 - 1$

C $2^5 - 1$

D $2^6 - 1$

E $2^7 - 1$

4. The mean, median and mode of the numbers in the boxes below are the same. What is the missing number?



A 6.5

B 7

C 8

D 8.5

E 9

5. A solid 'star' shape is created by gluing a square-based pyramid, in which each edge is of length 1 unit, precisely onto each face of a cube of edge 1 unit. How many faces does this 'star' have?

A 18

B 24

C 30

D 36

E 48

6. Harriet Hare and Turbo Tortoise want to cross the finish line together on their 12 mile woodland race. Turbo sets off at 8:15 am and trots at a constant speed of 4 mph. Given that Harriet runs at a constant speed of 8 mph, at what time should she set off?

A 9:45 am

B 10:15 am

C 10:45 am

D 11:15 am

E 11:45 pm

7. The Queen of Spades always lies for the whole day or always tells the truth for the whole day. Which of these statements can she never say?

A "Yesterday, I told the truth."

B "Yesterday, I lied."

C "Today, I tell the truth."

D "Today, I lie."

E "Tomorrow, I shall tell the truth."

8. Sydney flew to Melbourne, Australia. The flying time to Melbourne, which is 11 hours ahead of Britain, was 21 hours. Sydney's flight left London at 11.30am on Tuesday. What time was it in Melbourne when Sydney's flight arrived?

A 9:30pm on Tuesday

B 8:30am on Wednesday

C 7:30pm on Wednesday

D 6:30am on Thursday

E 7:30pm on Thursday

9. The diagram shows 10 identical coins which fit exactly inside a wooden frame. As a result each coin is prevented from sliding. What is the largest number of coins that may be removed so that each remaining coin is still unable to slide?

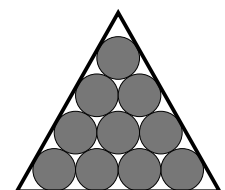
A 1

B 2

C 3

D 4

E 5



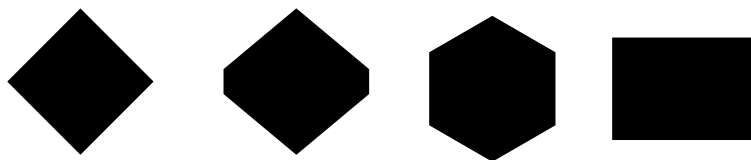
10. Gill is 18 this year. She and I went to a restaurant for lunch to celebrate her birthday. The bill for lunch for the two of us came to £25.50. Gill paid the bill by credit card and I left a £2.50 tip in cash. We agreed to split the total cost equally. How much did I owe Gill?

- A £11 B £11.50 C £12 D £12.50 E £13

11. What is the obtuse angle between the hands of a clock at 6 minutes past 8 o'clock?

- A 123° B 126° C 153° D 156° E 159°

12. When a solid cube is held up to the light, how many of the following shapes could its shadow have?

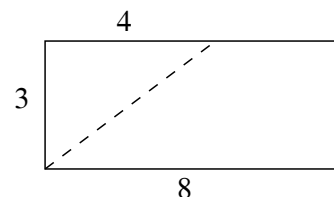


- A 0 B 1 C 2 D 3 E 4

13. What is 50% of 2006 plus 2006% of 50?

- A 1013.3 B 1053 C 1103.3 D 1504.5 E 2006

14. A 3×8 rectangle is cut into two pieces along the dotted line shown. The two pieces are then rearranged to form a right-angled triangle. What is the perimeter of the triangle formed?



- A 21 B 22 C 23 D 24 E 25

15. What is the mean of $1.\dot{2}$ and $2.\dot{1}$?

- A $1.\dot{6}$ B 1.666 C 1.665 D $1.6\dot{5}$ E 1.65



16. Al, Bertie, Chris and Di have sums of money totalling £150. Al and Bertie have £55 between them and Al and Chris have £65 between them. What is the difference between the amounts that Al and Di have?

- A £25 B £30 C £35 D £40 E £45

17. Last year, on the television programme *Antiques Roadshow*, a painting was said to be worth £15 000 although the painting had originally cost only 50p. As a percentage of the original price, what would be the approximate profit if the painting were to be sold for £15 000?

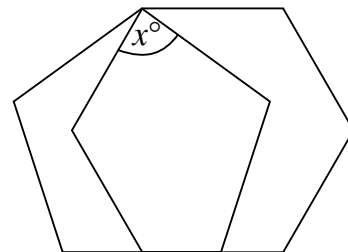
- A 15 000 % B 30 000 % C 300 000 % D 1 500 000 % E 3 000 000 %

18. Given that $4^x + 4^x + 4^x + 4^x = 4^{16}$, what is the value of x ?

- A 2 B 4 C 8 D 12 E 15

19. The diagram shows a regular pentagon and a regular hexagon which overlap. What is the value of x ?

A 82 B 84 C 85 D 87 E 91



20. Given that the number 2006 is the correct answer to the calculation

$$1 - 2 + 3 - 4 + 5 - 6 + \dots + (n - 2) - (n - 1) + n,$$

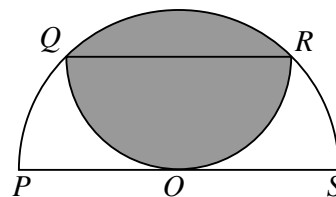
what is the sum of the digits of n ?

A 3 B 4 C 5 D 6 E 7

21. The diagram shows two semicircular arcs, $PQRS$ and QOR . The diameters, PS and QR , of the two semicircles are parallel; PS is of length 4 and is a tangent to semicircular arc QOR .

What is the area of the shaded region?

A $2\pi - 2$ B 3π C π D 4 E $2\pi - 4$



22. An 8 by 8 chessboard is placed so that a black square is in the top left-hand corner. Starting in the top left square and working along each row in turn, coloured counters are placed, one on each square, following the sequence black, white, red, black, white, red and so on. When the right-hand end of each row is reached, the pattern continues, starting at the left-hand end of the row beneath, until there is one counter on every square.

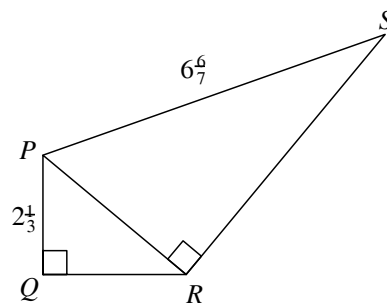
In the final arrangement, what fraction of the counters are on squares of the same colour as themselves?

A $\frac{11}{32}$ B $\frac{23}{64}$ C $\frac{7}{16}$ D $\frac{1}{2}$ E $\frac{2}{3}$

23. In the figure on the right, $PQ = 2\frac{1}{3}$, $PS = 6\frac{6}{7}$ and $\angle QPR = \angle RPS$.

How long is PR ?

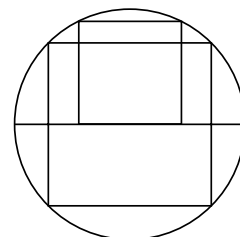
A $3\frac{1}{2}$ B 4 C $4\frac{1}{4}$ D $4\frac{25}{42}$ E 5



24. The diagram shows a square of area x square units inscribed inside a semicircle and a larger square of area y square units inscribed inside a circle.

What is the ratio $x : y$?

A $1 : \sqrt{2}$ B $1 : 2$ C $2 : 5$ D $1 : 3$ E $\sqrt{3} : 4$



25. Given that $5^j + 6^k + 7^l + 11^m = 2006$ where j, k, l and m are different non-negative integers, what is the value of $j + k + l + m$?

A 6 B 7 C 8 D 9 E 10